FACT SHEET: PROPERTIES OF ROOTS & RADICALS

Definitions:
- "$b = \sqrt[n]{a}$" means $b^n = a$. We say "*b* is the n^{th} root of *a*".
- In the expression $\sqrt[n]{a}$, *n* is the ***index***, and *a* is the ***radicand***.

Even index:

- If **radicand is negative**, the number is not real. **Ex.:** $\sqrt{-4}$ and $\sqrt[6]{-7}$ are not real #'s.
  (Can you think of a real number *n* such that $n^2 = -4$? No!)
- If the **radicand involves an unknown**, use absolute value bars in your answer if the answer calls for the principal root. **Ex.:** $\sqrt{(x+3)^2} = |x+3|$.
- To find the **domain of a function**, set the radicand greater than or equal to zero and solve. **Ex.:** $f(x) = \sqrt[4]{3-x} \rightarrow$ Set $3 - x \geq 0$ & solve for *x* to find domain.

Odd index:

- If the **radicand is negative**, the number is still real. **Ex.:** $\sqrt[5]{-32} = -2$.
  [Since $(-2)^5 = (-2)(-2)(-2)(-2)(-2) = -32$]
- If the **radicand involves an unknown**, absolute value bars are not necessary.
  Ex.: $\sqrt[7]{x^7} = x$, not $|x|$ (if *x* is negative, we want it to remain negative in the answer).
- The **domain of the function** is simply $\mathbb{R}$, the set of real numbers.
  Ex.: $g(x) = \sqrt[5]{3-x}$ has domain $\mathbb{R}$.

Rational Exponents:

- $a^{1/n} = \sqrt[n]{a}$
- $a^{m/n} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$

Ex.: $16^{1/4} = \sqrt[4]{16} = 2$

Ex.: $8^{2/3} = \left(\sqrt[3]{8}\right)^2 = 2^2 = 4$, or

$8^{2/3} = \sqrt[3]{8^2} = \sqrt[3]{64} = 4$

Multiplication and Division:

- $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$ [Note: Indexes must be alike! Multiply the radicands]
- $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\dfrac{a}{b}}$ $(b \neq 0)$ [Note: Indexes must be alike! Divide the radicands]

Ex.: $\sqrt[3]{5} \cdot \sqrt[3]{6} = \sqrt[3]{5 \cdot 6} = \sqrt[3]{30}$

Ex.: $\dfrac{\sqrt{98}}{\sqrt{2}} = \sqrt{\dfrac{98}{2}} = \sqrt{49} = 7$

Counterexample: $\sqrt[3]{10} \cdot \sqrt[4]{10} \neq \sqrt[12]{10}$

Addition and Subtraction: !!!!!!!!!! WARNING: $\sqrt[n]{a \pm b} \neq \sqrt[n]{a} \pm \sqrt[n]{b}$!!!!!!!!!

- You may only add/subtract *like radicals*, which have **same index and same radicand**.

  Ex.: $5 \cdot \sqrt[3]{xy} + 7 \cdot \sqrt[3]{xy} = 12 \cdot \sqrt[3]{xy}$ (add/subtract coefficients, as with polynomials)

  Counterex's: $\sqrt[3]{x} + \sqrt[4]{x} \neq \sqrt[7]{x}$; $\sqrt[4]{x} - \sqrt[4]{y} \neq \sqrt[4]{x - y}$

Rationalizing Denominators:

- To put $\frac{a}{\sqrt{b}}$ into proper form, multiply the top and bottom by $\sqrt{b}$:

  $\frac{a}{\sqrt{b}} = \frac{a}{\sqrt{b}} \cdot \frac{\sqrt{b}}{\sqrt{b}} = \frac{a\sqrt{b}}{b}$ **Ex.:** $\frac{3}{\sqrt{5}} = \frac{3}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$

- To put $\frac{a}{b + \sqrt{c}}$ into proper form, multiply top & bottom by the **conjugate** of $b + \sqrt{c}$:

  $\frac{a}{b + \sqrt{c}} = \frac{a}{(b + \sqrt{c})} \cdot \frac{(b - \sqrt{c})}{(b - \sqrt{c})}$ … and FOIL the denominators *carefully*.

Ex.: $\frac{15}{4+\sqrt{6}} = \frac{15}{4+\sqrt{6}} \cdot \frac{(4-\sqrt{6})}{(4-\sqrt{6})} = \frac{15(4-\sqrt{6})}{16 - 4\sqrt{6} + 4\sqrt{6} - 6} = \frac{15(4-\sqrt{6})}{10} = \frac{3(4-\sqrt{6})}{2} = \frac{12-3\sqrt{6}}{2}$

(or $6 - \frac{3}{2}\sqrt{6}$)

Principle of Powers:

- If $a = b$, then $a^n = b^n$.

- Steps for solving a **radical equation**:

1) **Isolate the radical** on one side of the equation.

2) Use the **principle of powers** to eliminate the radical (if the radical is an n^{th} root, raise both sides of the equation to the n^{th} power).

3) Solve the resulting equation.

4) **Always check** for extraneous (false) solutions!

In this case, the solution $x = 13$ is true. If it were false, we would eliminate the answer and say "no solution".

Ex.: $\sqrt{x - 4} + 7 = 10$

$\quad\quad\quad\quad -7 \quad -7$

1) $\sqrt{x - 4} = 3$

2) $\left(\sqrt{x - 4}\right)^2 = 3^2$

$\longrightarrow \; x - 4 = 9$

3) $\quad +4 \quad +4$

$\longrightarrow \; x = 13$

4) Check: $\sqrt{13 - 4} + 7 = 10$?

$\sqrt{9} + 7 = 10$?

$3 + 7 = 10$ ✓