Imperial Valley College
Math Lab

QUADRATICS SUMMARY SHEET (EQUATIONS, FUNCTIONS, AND GRAPHS)

The two basic forms of **quadratic equations** are: $y = ax^2 + bx + c$ $y = a(x-h)^2 + k$
The two basic forms of **quadratic functions** are: $f(x) = ax^2 + bx + c$ $f(x) = a(x-h)^2 + k$

SQUARE ROOT PROPERTY

If $X^2 = k$, then $X = \pm\sqrt{k}$. [can be any algebraic expression, and k is any real number].

Ex: $(x-4)^2 = 5 \quad \rightarrow \quad x - 4 = \pm\sqrt{5} \quad \rightarrow \quad x = 4 \pm \sqrt{5}$

QUADRATIC FORMULA

If $ax^2 + bx + c = 0$, then $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$.

Remark: Notice that the equation *must* be set equal to zero first!

Ex: $4x^2 + 5x - 3 = 0$ First note that $a = 4$, $b = 5$, and $c = -3$.

Substitute *carefully*: $x = \frac{-5 \pm \sqrt{5^2-4(4)(-3)}}{2(4)} = \frac{-5 \pm \sqrt{25-(-48)}}{8} = \frac{-5 \pm \sqrt{73}}{8}$.

Watch the signs! Especially when *a* or *c* is negative.

THE DISCRIMINANT

The type of solution to a quadratic equation depends upon what happens under the radical sign $\left(\sqrt{\quad}\right)$. Therefore, if you look at the Quadratic Formula above, you'll see that the "$b^2 - 4ac$" part is important.

Definition: For the quadratic equation $ax^2 + bx + c = 0$, the ***discriminant*** is $b^2 - 4ac$.

If $b^2 - 4ac > 0$ and is a perfect square, then $ax^2 + bx + c = 0$ has **two rational solutions**.

If $b^2 - 4ac > 0$ but is not a perfect square, then $ax^2 + bx + c = 0$ has **two irrational solutions**.

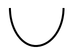
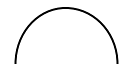
If $b^2 - 4ac = 0$, then $ax^2 + bx + c = 0$ has **one rational solution**.

If $b^2 - 4ac < 0$, then $ax^2 + bx + c = 0$ has **two complex solutions** (involving the imaginary number).

*For our purposes here, *irrational solutions* involve square roots that are real but cannot be simplified, such as $\sqrt{3}$ or $\sqrt{11}$, while *rational solutions* no longer have square root signs after simplification, e.g. $\sqrt{9} = 3$. The example above has two irrational solutions because of the $\pm\sqrt{73}$.

GRAPHING QUADRATIC EQUATIONS

The graphing method depends on whether you have the form $f(x) = ax^2 + bx + c$ or $f(x) = a(x-h)^2 + k$.
If $a \neq 0$, the graph will always be a **parabola**. In both cases…

1) If $a > 0$, then parabola opens upward.  If $a < 0$, the parabola opens downward. 

2) Find the vertex. a. For $f(x) = a(x-h)^2 + k$, the vertex is (h, k).

   b. For $f(x) = ax^2 + bx + c$, the vertex is $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$.

3) Find and plot the *y*-intercept by setting $x = 0$ and computing $f(x)$.

4) Find and plot the *x*-intercept(s) by setting $f(x) = 0$ (i.e. setting $y = 0$) and finding the *x*-value(s).

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Example: Graph $f(x) = -(x+1)^2 + 4$ and $f(x) = -x^2 - 2x + 3$:
[You should verify that these are actually two different forms of the same function]
[Please see a tutor for guidance through this example if necessary]

1) Since $a = -1$ is negative, the parabola will open downward.

2) a. The vertex $(h, k) = (-1, 4)$. Why does $h = -1$? Notice in the general formula, the ***h* must follow the minus sign**. So, think of $(x+1)^2$ as $(x - -1)^2$. This means $h = -1$.

   b. $h = \frac{-b}{2a} = \frac{-(-2)}{2(-1)} = \frac{2}{-2} = -1$ (just as it did above), and
   $k = f\left(\frac{-b}{2a}\right) = f(-1) = -(-1)^2 - 2(-1) + 3 = -1 + 2 + 3 = 4$ (just as it did above).

3) Substituting $x = 0$, we obtain $f(0) = 3$. So, the *y*-intercept is $(0, 3)$.

4) Set $f(x) = 0$ and solve for x. For practice, check both cases:

$$0 = -(x+1)^2 + 4 \quad \text{and} \quad 0 = -x^2 - 2x + 3$$

In both cases, you should get $x = -3$ and $x = 1$. So the *x*-intercepts are $(-3, 0)$ and $(1, 0)$.