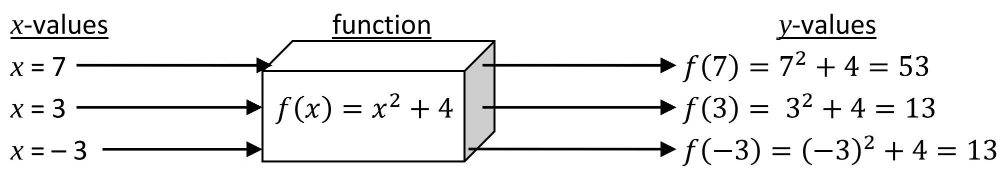
Imperial Valley College
Math Lab

FUNCTIONS (Definitions and Basics)

A **function** is a rule that assigns each x-value to one unique y-value.
The set of all x-values being used is called the **domain**; the set of all corresponding y-values is called the **range**.

A function may be thought of as a machine, in which you input x-values, and the machine outputs y-values.

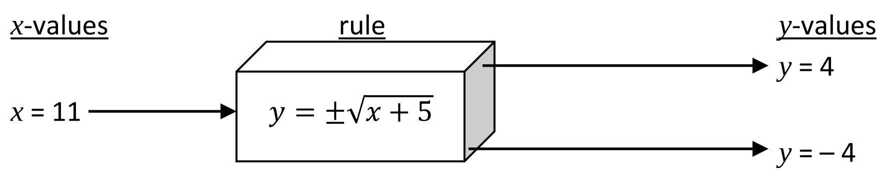
EXAMPLE:



IMPORTANT: **It is possible to have more than one x-value correspond to the same y-value.**
In the above example, both $x = 3$ and $x = -3$ correspond to $y = 13$.

HOWEVER, it is NOT possible for one x-value to correspond to more than one y-value.
If this happens, then the rule is not a function (it can be called a relation).

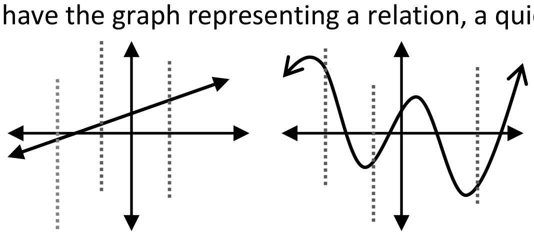
EXAMPLE:



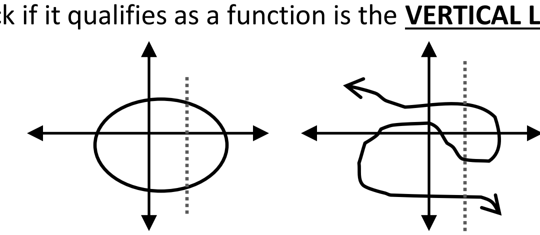
The above example is not a function,
since $x = 11$ corresponds to both $y = 4$ and $y = -4$.

VERTICAL LINE TEST

If you have the graph representing a relation, a quick way to check if it qualifies as a function is the **VERTICAL LINE TEST**:



FUNCTIONS



NOT FUNCTIONS

If each x-value has only one y-value corresponding to it, then you can draw a vertical line through *any* part of the graph, and it will only cross through the graph *one time*.

These pass the vertical line test, so they're functions.

If there is an x-value with *more than one y-value* corresponding to it, then a vertical line through that x-value will *cross the graph more than one time*.

These fail the vertical line test, so they're not functions.

OPERATIONS ON FUNCTIONS

Functions can be added, subtracted, multiplied, and divided. The shorthand notation is provided in the example below:

EXAMPLE: $f(x) = 2x + 10$ $g(x) = x^2 - 6$

Operation	**Notation**		**Interpretation**		**Computation/Simplification**		
Addition:	$(f+g)(x)$	=	$f(x)+g(x)$	=	$(2x+10)+(x^2-6)$	=	x^2+2x+4
Subtraction:	$(f-g)(x)$	=	$f(x)-g(x)$	=	$(2x+10)-(x^2-6)$	=	$-x^2+2x+16$
Multiplication:	$(f\cdot g)(x)$	=	$f(x)\cdot g(x)$	=	$(2x+10)(x^2-6)$	=	$2x^3+10x^2-12x-60$
Division:	$\left(\frac{f}{g}\right)(x)$	=	$\frac{f(x)}{g(x)}$	=	$\frac{2x+10}{x^2-6}$		

PROBLEMS INVOLVING FUNCTION NOTATION

The expression "$f(x) = y$" is pronounced "f of x equals y", and it indicates that x **is the input** and y **is the output**.

EXAMPLE: Suppose $f(x) = -2x + 3$. a) $f(-1) = ?$ b) Find x such that $f(x) = -1$

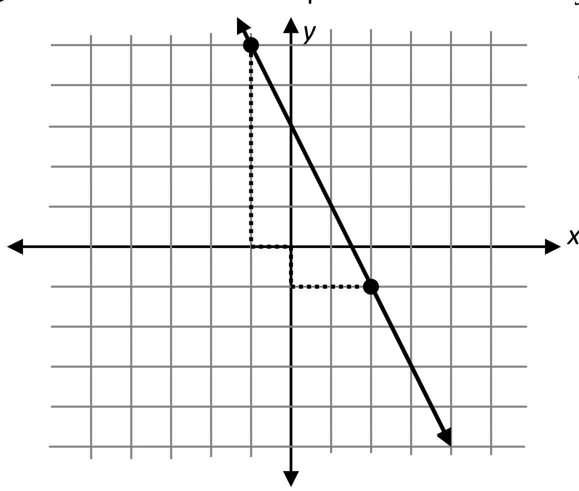
a) **Notation**: $f(-1) =$ _____

Interpretation: "When $x = -1$, what is y?"

Evaluation: Substitute $x = -1$ into the function's expression, then compute

$$f(x) = -2x + 3$$
$$f(-1) = -2(-1) + 3$$
$$= 2 + 3$$
$$= 5.$$

Graph: Locate $x = -1$ on the x-axis, then locate the corresponding y-value directly above or below it (above in this example, at $y = 5$).



b) **Notation**: $f(x) = -1$

Interpretation: "When $y = -1$, what is x?"

Evaluation: Replace the $f(x)$ with the value it is equal to. Solve the resulting equation for x.

$$f(x) = -2x + 3$$
$$-1 = -2x + 3 \quad \rightarrow \quad \begin{array}{rl} -1 = & -2x + 3 \\ \underline{-3} & \underline{\quad\quad -3} \\ -4 = & -2x \end{array} \quad \longrightarrow \quad x = 2.$$

Graph: Locate $y = -1$ on the y-axis. Then find the corresponding x-value directly to the left or right (right, in this example, at $x = 2$).

Check: $f(2) = -2(2) + 3 = -4 + 3 = -1$ ✓