ABSOLUTE VALUE BARS IN EQUATIONS AND INEQUALITIES

In the following four rules, X and Y represent any algebraic expressions that may be inside the absolute value bars (not necessarily just a single variable). Also, "*a*" is assumed to be a *positive* number.

1) $|X| = a$ means $X = a$ or $X = -a$

Ex. $|2x-4| = 10 \longrightarrow 2x - 4 = 10$ or $2x - 4 = -10$

Now solve each equation separately.

2) $|X| = |Y|$ means $X = Y$ or $X = -Y$

Ex. $|3x+2| = |4x-9| \longrightarrow 3x + 2 = 4x - 9$ or $3x + 2 = -(4x - 9)$

Now solve each equation separately.
Notice that in the equation on the right, the *entire expression* $4x - 9$ was negated. You must remember to **distribute the negative sign**:

$$-(4x - 9) = -4x + 9$$

3) $|X| \le a$ means $-a \le X \le a$

Ex. $|7x+4| \le 30 \longrightarrow -30 \le 7x + 4 \le 30$

4) $|X| \ge a$ means $X \ge a$ or $X \le -a$

Ex. $|5x| \ge 11 \longrightarrow 5x \ge 11$ or $5x \le -11$

REMEMBER: If necessary, *isolate the absolute value bars* first before applying the above rules!

Ex. $2|x-4| + 3 = 9 \quad \rightarrow$

$$\begin{array}{rl} 2|x-4| + 3 &= 9 \\ -3 &\;\; -3 \\ \hline 2|x-4| &= 6 \end{array}$$

$$\rightarrow \quad \frac{2|x-4|}{2} = \frac{6}{2}$$

$$\rightarrow \quad |x-4| = 3$$

Now apply Rule (1) from above.

REMEMBER: Rules (1) and (3) only make sense if *a* is positive. Otherwise, "no solution".
In Rule (4), if *a* is negative, then the inequality is *always true* (no work needed!).

Ex's $|2x+1| = -5$ and $|3x-9| \le -4$ have **no solutions.** $|x-1| > -2$ is **always true**.