

PROPERTIES of REAL NUMBERS (\mathbb{R})

Commutative Property of Addition & Multiplication

- **Commutative Properties** state that the order in which two real numbers are added or multiplied does not affect their sum or product.

For all real numbers a & b ...

Addition:

$$a + b = b + a$$

Multiplication:

$$ab = ba$$

Ex.: If $a = 2$ & $b = 3$, then...

$$2 + 3 = 3 + 2$$

$$2 \cdot 3 = 3 \cdot 2$$

$$5 = 5$$

(TRUE)

Statement(s)

$$6 = 6$$

(TRUE)

Associative Property of Addition & Multiplication

- **Associative Properties** state that regrouping numbers that are added or multiplied does not affect their sum or product.

For all real numbers a , b , & c ...

Addition:

$$(a + b) + c = a + (b + c)$$

Multiplication:

$$(ab)c = a(bc)$$

Ex.: If $a = 2$, $b = 3$ & $c = 4$, then...

$$(2 + 3) + 4 = 2 + (3 + 4)$$

$$(2 \cdot 3) \cdot 4 = 2 \cdot (3 \cdot 4)$$

$$(5) + 4 = 2 + (7)$$

$$9 = 9$$

(TRUE)

Statement(s)

$$(6) \cdot 4 = 2 \cdot (12)$$

$$24 = 24$$

(TRUE)

CAUTION: Commutative & Associative Properties are Not Applicable under Subtraction and Division Operations.

Check the next **Counterexamples** using the real numbers $a = 8$, $b = 4$ and $c = 2$:

Commutative Property for...

Associative Property for...

Subtraction:

Division:

Subtraction:

Division:

$$a - b \neq b - a$$

$$a \div b \neq b \div a$$

$$(a - b) - c \neq a - (b - c)$$

$$(a \div b) \div c \neq a \div (b \div c)$$

$$8 - 4 \stackrel{?}{=} 4 - 8$$

$$8 \div 4 \stackrel{?}{=} 4 \div 8$$

$$(8 - 4) - 2 \stackrel{?}{=} 8 - (4 - 2)$$

$$(8 \div 4) \div 2 \stackrel{?}{=} 8 \div (4 \div 2)$$

$$4 \neq -4$$

$$2 \neq 0.5$$

$$(4) - 2 \stackrel{?}{=} 8 - (2)$$

$$(2) \div 2 \stackrel{?}{=} 8 \div (2)$$

(FALSE) Statements (FALSE)

$$2 \neq 6$$

$$1 \neq 4$$

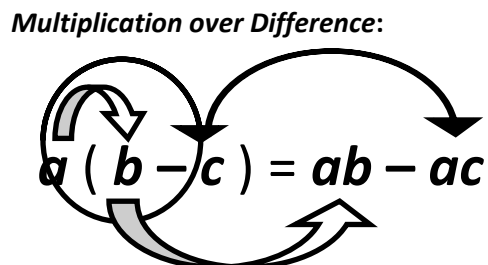
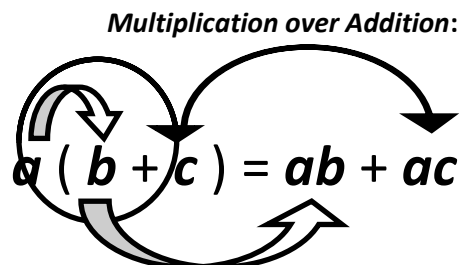
(FALSE) Statements (FALSE)

PROPERTIES of REAL NUMBERS (\mathbb{R})

Distributive Property of *Multiplication* over *Addition*

- **Distributive Property** states that multiplication distributes over addition or difference between two or more terms.

For all real numbers a , b , and c , distribute or multiply factor a , as a common factor, over the sum or difference of the terms b & c ...



Remember that Condition on Subtraction:

$$(b-c) \neq (c-b).$$

Ex.: If $a = 2$, $b = 3$, & $c = 4$, then...

LHS	RHS	LHS	RHS
By Adding the like terms inside parentheses.	By distributing the factor 2 over the sum inside parentheses.	By subtracting the like terms inside parentheses.	By distributing the factor 2 over the difference inside parentheses.
$2(3+4)$	$= 2(3) + 2(4)$	$2(3-4)$	$= 2(3) - 2(4)$
$2(7)$	$= 6 + 8$	$2(-1)$	$= 6 - 8$
14	$= 14$	-2	$= -2$
(TRUE)		(TRUE)	

Statement(s)

Identity Property of *Addition* & *Multiplication*

- **Identity Properties** state that when adding or multiplying a real number, the result is that same real number. For all real numbers a ...

In Addition, the Additive Identity is **ZERO**:

$$a + 0 = 0 + a = a$$

In Multiplication, the Multiplicative Identity is **ONE**:

$$a \cdot 1 = 1 \cdot a = a$$

Inverse Property of *Addition* & *Multiplication*

- **Inverse Properties** state that when adding or multiplying a real number, the result is equal to such **Identity Number**, **ZERO** for **Addition** and **ONE** for **Multiplication**. For all real numbers a , except 0 for multiplication...

In Addition, the Additive Inverse or Opposite of a is $(-a)$:

$$a + (-a) = (-a) + a = 0.$$

In Multiplication, the Multiplicative Inverse or Reciprocal of a is $\frac{1}{a}$, and $a \neq 0$: $a \cdot \frac{1}{a} = \frac{1}{a} \cdot a = 1.$