PROPERTIES of REAL NUMBERS ($\mathbb{R}$)

Commutative Property of *Addition* & *Multiplication*

- **Commutative Properties** state that the order in which two real numbers are added or multiplied does not affect their sum or product.

For all real numbers ***a*** & ***b***…

Addition:

$$a + b = b + a$$

Multiplication:

$$ab = ba$$

Ex.: If ***a*** = 2 & ***b*** = 3, then…

Addition		Multiplication
2 + 3 = 3 + 2		2 • 3 = 3 • 2
5 = 5		6 = 6
(**TRUE**)	**Statement(s)**	(**TRUE**)

Associative Property of *Addition* & *Multiplication*

- **Associative Properties** state that regrouping numbers that are added or multiplied does not affect their sum or product.

For all real numbers ***a***, ***b***, & ***c***…

Addition:

$$(a + b) + c = a + (b + c)$$

Multiplication:

$$(ab)c = a(bc)$$

Ex.: If ***a*** = 2, ***b*** = 3 & ***c*** = 4, then…

Addition		Multiplication
(2 + 3)+ 4 = 2 +(3 + 4)		(2•3)•4 = 2•(3•4)
(5)+ 4 = 2 +(7)		(6)• 4 = 2•(12)
9 = 9		24 = 24
(**TRUE**)	**Statement(s)**	(**TRUE**)

CAUTION: **Commutative** & **Associative Properties** are Not Applicable under *Subtraction* and *Division* Operations.

Check the next ***Counterexamples*** using the real numbers ***a*** = 8, ***b*** = 4 and ***c*** = 2:

Commutative Property for…

Subtraction:	*Division*:
$a - b \neq b - a$	$a \div b \neq b \div a$
$8 - 4 \stackrel{?}{=} 4 - 8$	$8 \div 4 \stackrel{?}{=} 4 \div 8$
$4 \neq -4$	$2 \neq 0.5$
(***FALSE***)	(***FALSE***)

Statements

Associative Property for…

Subtraction:	*Division*:
$(a - b) - c \neq a - (b - c)$	$(a \div b) \div c \neq a \div (b \div c)$
$(8 - 4) - 2 \stackrel{?}{=} 8 - (4 - 2)$	$(8 \div 4) \div 2 \stackrel{?}{=} 8 \div (4 \div 2)$
$(4) - 2 \stackrel{?}{=} 8 - (2)$	$(2) \div 2 \stackrel{?}{=} 8 \div (2)$
$2 \neq 6$	$1 \neq 4$
(***FALSE***)	(***FALSE***)

Statements

PROPERTIES of REAL NUMBERS ($\mathbb{R}$)

Distributive Property of *Multiplication* over *Addition*

- **Distributive Property** states that multiplication distributes over addition or difference between two or more terms.

For all real numbers ***a***, ***b***, and ***c***, distribute or multiply factor ***a***, as a common factor, over the sum or difference of the terms ***b*** & ***c*** …

***Multiplication over Addition*:**

$$a\,(\,b + c\,) = ab + ac$$

***Multiplication over Difference*:**

$$a\,(\,b - c\,) = ab - ac$$

***Remember that Condition on Subtraction*:**

(b – c) ≠ (c – b).

Ex*.:** If ***a = 2, ***b*** = 3, & ***c*** = 4, then…

LHS		***RHS***
By Adding the like terms inside parentheses.		By distributing the factor 2 over the sum inside parentheses.
2(3 + 4)	**=**	2**(** 3 **)** **+** 2**(** 4 **)**
2**(** 7 **)**	**=**	6 **+** 8
14	**=**	14

(TRUE)

LHS		***RHS***
By subtracting the like terms inside parentheses.		By distributing the factor 2 over the difference inside parentheses.
2(3 – 4)	**=**	2**(** 3 **)** **–** 2**(** 4 **)**
2**(–** 1 **)**	**=**	6 **–** 8
– 2	**=**	**–** 2

(TRUE)

Statement(s)

Identity Property of *Addition* & *Multiplication*

- **Identity Properties** state that when adding or multiplying a real number, the result is that same real number. For all real numbers a…

In Addition, the Additive Identity is **ZERO**:

$$a + \mathbf{0} = \mathbf{0} + a = a$$

In Multiplication, the Multiplicative Identity is **ONE**:

$$a \bullet \mathbf{1} = \mathbf{1} \bullet a = a$$

Inverse Property of *Addition* & *Multiplication*

- **Inverse Properties** state that when adding or multiplying a real number, the result is equal to such **<u>Identity</u> Number**, **<u>ZERO</u> for Addition** and **<u>ONE</u> for Multiplication**. For all real numbers a, except 0 for multiplication…

In Addition, the Additive Inverse or Opposite of a is ($\boldsymbol{-a}$): $a + (\,\boldsymbol{-a}) = (\,\boldsymbol{-a}) + a = 0$.

In Multiplication, the Multiplicative Inverse or Reciprocal of a is $\frac{\mathbf{1}}{\boldsymbol{a}}$, and $a \neq 0$: $a \bullet \frac{\mathbf{1}}{\boldsymbol{a}} = \frac{\mathbf{1}}{\boldsymbol{a}} \bullet a = 1$.