FACTS ABOUT LOGARITHMS

DEFINITION: For $a > 0$, $a \neq 1$, $log_a x = y$ means $a^y = x$.

In words, $f(x) = \log_a x$ is the *inverse function* of $f(x) = a^x$.

INVERSE FUNCTIONS: First, $f^{-1}(x) \neq \frac{1}{f(x)}$ (common misconception).

Rather, $f^{-1}(x)$ is the *inverse function* of $f(x)$, which "reverses" everything that $f(x)$ does.

FACT: $f^{-1}\left(f(x)\right) = f\left(f^{-1}(x)\right) = x$ for all x in the respective domains.

OTHER DEFINITIONS: The **common log** is $\log_{10} x$, usually written as "**log *x***"

The **natural log** is $\log_e x$ = "**ln(*x*)**" ($e \approx 2.71828$)

PROPERTIES OF LOGARITHMS:

i) $\log_a 1 = 0$ [this should be common sense, since $a^0 = 1$]

ii) $\log_a a = 1$ [this should be common sense, since $a^1 = a$]

iii) $\log_a a^m = m$ [cancellation property]

iv) $a^{\log_a m} = m$ [cancellation property]

v) $\log_a (xy) = \log_a x + \log_a y$ [Careful – see warning below]

vi) $\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$ [Careful – see warning below]

vii) $\log_a x^r = r \cdot \log_a x$ ["Bring the exponent down in front"]

WARNINGS ABOUT PROPERTIES (V) AND (VI):

$\log_a (x + y) \neq \log_a x + \log_a y$ [It does not work like the distributive law].

Also, $\frac{\log_a x}{\log_a y} \neq \log_a (x - y)$ and $\frac{\log_a x}{\log_a y} \neq \log_a \left(\frac{x}{y}\right)$. Basically, you can't simplify $\frac{\log_a x}{\log_a y}$.

<u>**Some Practice with Logarithms**</u>

1) $\log_{38} 38 =$ ______________

2) $52^{\log_{52} 9} =$ ______________

3) $\log_{64} 8 =$ ______________

4) $\log_5 \dfrac{1}{25} =$ ______________

5) $\log_3 \sqrt[4]{\dfrac{1}{3}} =$ ______________

6) $\ln e^{3a^2} =$ ______________

7) Write as a single log and simplify:
$\log_b(x^2 - 25) - \log_b(x - 5)$

8) Find all x such that $81^{x-1} = 27^{2x}$:

9) The logarithmic form of $\sqrt[5]{32} = 2$ is written as:

a. $\log_{32} 2 = \dfrac{1}{5}$

b. $\log_{32} \dfrac{1}{5} = 2$

c. $\log_2 32 = \dfrac{1}{5}$

d. $\log_2 \dfrac{1}{5} = 32$

e. $\log_{1/5} 32 = 2$

f. none of these

10) Using properties of logarithms, $\log_a \sqrt[3]{4x} =$

a. $\sqrt[3]{\log_a 4} + \sqrt[3]{\log_a x}$

b. $\log_a \sqrt[3]{4} \cdot \log_a \sqrt[3]{x}$

c. $\dfrac{1}{3}\left[\log_a 4 \cdot \log_a x\right]$

d. $\dfrac{1}{3}\left[\log_a 4 + \log_a x\right]$

e. none of these

11) Using properties of logarithms, $\log_5\left(x^2 + 4x\right) =$

a. $\log_5 x^2 + \log_5 4x$

b. $2 \cdot \log_5 (5x)$

c. $\log_5 x^2 \cdot \log_5 4x$

d. none of these

12) Find x such that
$\log_3(x^2 + 17) - \log_3(x + 5) = 1$:

<u>**CHALLENGE**</u>:
If $\log_m 2 = p$ and $\log_m 3 = r$,
find $\log_m(144m^7)$.