CONIC SECTIONS

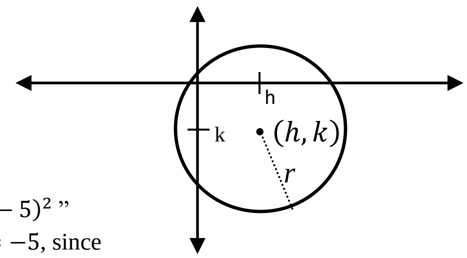
CIRCLE

General Form: $x^2 + y^2 + Dx + Ey + F = 0$

*To graph, use the "completing the square" method to convert from **general form** to **standard form**:*

Standard Form: $(\boldsymbol{x} - \boldsymbol{h})^2 + (\boldsymbol{y} - \boldsymbol{k})^2 = \boldsymbol{r}^2$

Center = (h, k); Radius = r



NOTE: h and k are being subtracted from x and y. For example, "$(x-5)^2$" would indicate that $h = 5$, while "$(x+5)^2$" would indicate that $h = -5$, since $(x+5)^2 = (x-(-5))^2$.

PARABOLA

*To sketch a parabola, you must determine where the **vertex** $(\boldsymbol{h}, \boldsymbol{k})$ is located. The equation of a parabola comes in two different common forms, and the procedure for finding the **vertex** depends upon which form of equation you have.*

Vertical Parabola:

$y = a(x-h)^2 + k$

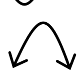
Vertex: (h, k)

$y = ax^2 + bx + c$

Vertex:

1) Find the x-value by computing $x = -\frac{b}{2a}$
2) Find the y-value by substituting this new x-value back into the equation

Orientation:

- If $a > 0$, the parabola opens upward: 
- If $a < 0$, the parabola opens downward: 

Horizontal Parabola:

$x = a(y-k)^2 + h$

Vertex: (h, k)

$x = ay^2 + by + c$

Vertex:

1) Find the y-value by computing $y = -\frac{b}{2a}$
2) Find the x-value by substituting this new y-value back into the equation

Orientation:

- If $a > 0$, the parabola opens to the right: 
- If $a < 0$, the parabola opens to the left: 

Intercepts:

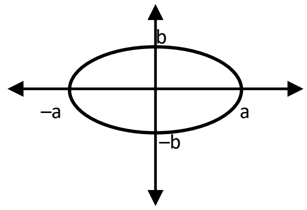
In all cases above, the following rules apply:

- To find the x-intercept(s), set $y = 0$ and solve for x.
- To find the y-intercept(s), set $x = 0$ and solve for y.

ELLIPSE

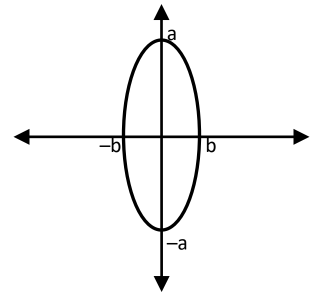
[CENTERED AT THE ORIGIN, AND ASSUMING $a^2 > b^2$]

Horizontal, with vertices $(a, 0)$ and $(-a, 0)$:



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Vertical, with vertices $(0, a)$ and $(0, -a)$:



$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

TIP: It may be helpful to remember that the number under the " x^2 " always corresponds to *how far left and right* you move on the x-axis, while the number under the " y^2 " always corresponds to *how far up and down* you move on the y-axis.

SHIFTED ELLIPSE

[ASSUMING $a^2 > b^2$]

Horizontal, with vertices $(a, 0)$ and $(-a, 0)$:

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

Vertical, with vertices $(0, a)$ and $(0, -a)$:

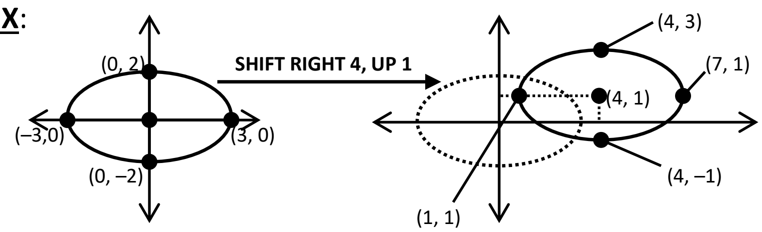
$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

Both of these equations represent ellipses that are **centered at** $(\boldsymbol{h}, \boldsymbol{k})$.

In other words, if the ellipse is initially drawn so that it's centered at the origin, it would be shifted as follows:

- $\boldsymbol{h}$ units to the right if $\boldsymbol{h}$ is positive
- $\boldsymbol{h}$ units to the left if $\boldsymbol{h}$ is negative
- $\boldsymbol{k}$ units upward if $\boldsymbol{k}$ is positive
- $\boldsymbol{k}$ units downward if $\boldsymbol{k}$ is negative

EX:



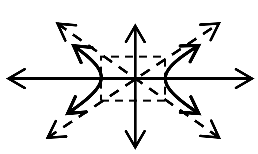
$\frac{x^2}{9} + \frac{y^2}{4} = 1$ **SHIFT** → $\frac{(x-4)^2}{9} + \frac{(y-1)^2}{4} = 1$

HYPERBOLA

[CENTERED AT THE ORIGIN]

Hyperbola with x-intercepts $(a, 0)$ and $(-a, 0)$:

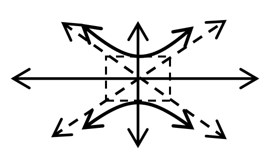
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$



Hyperbola with x-intercepts

Hyperbola with y-intercepts $(0, a)$ and $(0, -a)$:

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$



Hyperbola with y-intercepts

TIP: Notice that when the " x^2 " comes first, the hyperbola has x-intercepts, and when the " y^2 " comes first, the hyperbola has y-intercepts.